Robust Sequence Recognition in Random Neuronal Networks

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Extended Abstract

Motivation. Sequence recognition, a fundamental working memory computation, can be realized in recurrent neuronal networks as input-driven transitions between near-critical discrete attractors (fixed-point equilibria) (Figure 1). Achieving such a dynamical landscape, where each attractor is poised near the edge of instability, conventionally poses a fragile parameter fine-tuning problem. However, recent work demonstrates that training biologically plausible excitatory-inhibitory (E/I) networks with additive background noise naturally circumvents this issue [1]. Such training organically produces a *sparse inhibitory core-excitatory periphery* structural motif coupled with slow, heterogeneous inhibitory timescales, leading to improved sequence recognition task performance attributed to emergent marginally stable equilibria. Yet, the fundamental mechanism by which these specific emergent network characteristics naturally push the equilibria toward marginal stability without manual fine-tuning remains to be understood.

Approach and Methodology. To analytically investigate how these emergent network properties shape near-marginal dynamical attractor landscapes, we analyze the local Jacobian matrices $J = \mathcal{T}^{-1}(-I + W\mathcal{H}^{*})$ governing small fluctuations around typical equilibria. Guided by post-training statistics, we formulate a random matrix ensemble where W is a sparse, non-Hermitian, rectangular-block matrix reflecting the pruned excitatory outputs and the sparse inhibitory core. This ensemble incorporates empirical distributions of heterogeneous synaptic decay timescales ($\mathcal{T}$) and activation-function gains at typical equilibria ($\mathcal{H}^{*}$). Because standard dense random matrix analytical techniques fail to accommodate network sparsity combined with block structure and timescale heterogeneity, we develop a novel sparse non-Hermitian random matrix theory using statistical field theory methods [2]. We evaluate a Hermitized resolvent representation of the spectral density using supersymmetric (SUSY) field integration in the style of Fyodorov and Mirlin, formulating large-N saddle-point equations to solve for the spectrum.

Results. By evaluating these saddle-point equations, we obtain an analytic description of the spectral edge, relating statistical parameters of the Jacobians (sparsity, weight variances, E/I ratio, and the distributions of timescales and gains) to near-critical features of the equilibria. These results reveal that the slow, inhibitory core naturally pushes the equilibria of these cortical networks toward marginal stability without requiring parameter fine-tuning.

Conclusions and Outlook. Our analytical edge condition resolves the fine-tuning paradox in working memory computation: the noise-induced emergence of the slow inhibitory core inherently shapes a favorable near-marginal dynamical landscape, guaranteeing that the

equilibria of these heterogeneous cortical networks are poised near the edge of instability. Additional studies will analyze the Inverse Participation Ratios (IPR) of the corresponding left-eigenvectors to elucidate how non-normality and transient amplification facilitate the rapid, stimulus-driven hops between these memory attractors. Furthermore, the amount of noise required to sustain the non-equilibrium steady states (NESS) near these typical discrete attractors is related to the entropy production, a fundamental quantity in non-equilibrium statistical mechanics. In this context, we will also discuss a newly discovered universal entropy production rate formula for various directed networks [3].

References

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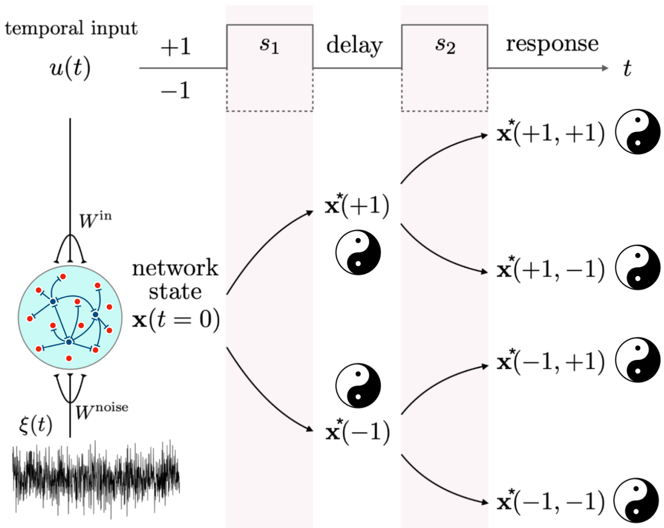

Figure 1: **Sequence recognition via discrete attractor hopping.** A recurrent neuronal network driven by sequential inputs (s_1, s_2) and additive noise $\xi(t)$ performs working memory tasks by hopping between stimulus-specific discrete attractors (fixed-point equilibria, $\mathbf{x}^*$). To maintain memory traces during delays while remaining responsive to new input drives, *all* intermediate operating equilibria must be near-marginally stable. Achieving this edge of instability conventionally requires fragile parameter fine-tuning. However, training with additive noise naturally circumvents this by producing a *sparse inhibitory core-excitatory periphery motif coupled with slow inhibitory dynamics*. Crucially, how these emergent characteristics robustly ensure near-marginal stability without fine-tuning can only be understood analytically through the sparse non-Hermitian random matrix theory developed here.