Rising inequality and a 'power shift' from the core to a parasitic periphery are precursors of collapse

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Keywords: Graph evolution, collapse, inequality, core-periphery, early-warning signals

Extended Abstract

Motivation. Understanding why collapses occur in complex systems is important for forecasting and response planning. Complex systems can appear stable for long periods yet collapse abruptly, making it important to understand how fragility accumulates over time. Tackling this problem for real systems is a difficult task; we explore this question through a mathematical model [1] which shows evolution of graph complexity. Previous studies have identified proximate causes of collapse in the model [2, 3, 4]. In particular, they have shown that at a collapse, the network is fragile in the sense that the number of redundant positive feedback pathways in the largest strong component that is the core of the graph becomes small. However, the long-term dynamics that causes this fragility to arise remains unclear.

Approach and Methodology. The model describes the evolution of a directed graph of $S \gg 1$ nodes. At any time, the eigenvector centrality X_i of node i is considered to be the 'strength' of the node. The network evolves one node at a time by replacing the weakest node by a randomly and sparsely connected node (having a connection probability p with other nodes such that $pS < 1$). The model exhibits spontaneous growth, stasis and collapse wherein the number of nodes with a nonzero strength first grows from order unity to reach its maximum possible value, S, remains there for a certain time, and then suddenly drops by a significant fraction of S in a single time step. To study the long-term evolution toward fragility, we compute the Gini coefficient, a measure of wealth inequality used in economics, for the distribution of node strengths X_i as a function of time, and track the average strengths of core and periphery nodes separately.

Results. We find that inequality in node strengths rises as the network approaches collapse (see Fig. 1, inset). Further, the average strength of periphery nodes, which are parasites in the sense that they do not feed back into the core that sustains them, becomes larger than the average strength of core nodes as a collapse is approached (Fig. 1). We refer to this phenomenon as a 'power shift' from the core to the periphery. We also find that a few periphery nodes that acquire a disproportionately large strength are the primary source of rising inequality. We thus identify rising inequality and power shift as two early warning signatures of collapse.

Conclusions and Outlook. These results show that the long-term route to fragility in the model is driven by a systematic redistribution of strength from the stabilizing core to a parasitic periphery. These lessons from the model could be relevant for social and economic systems, particularly in understanding the role of power redistribution in societal collapse.

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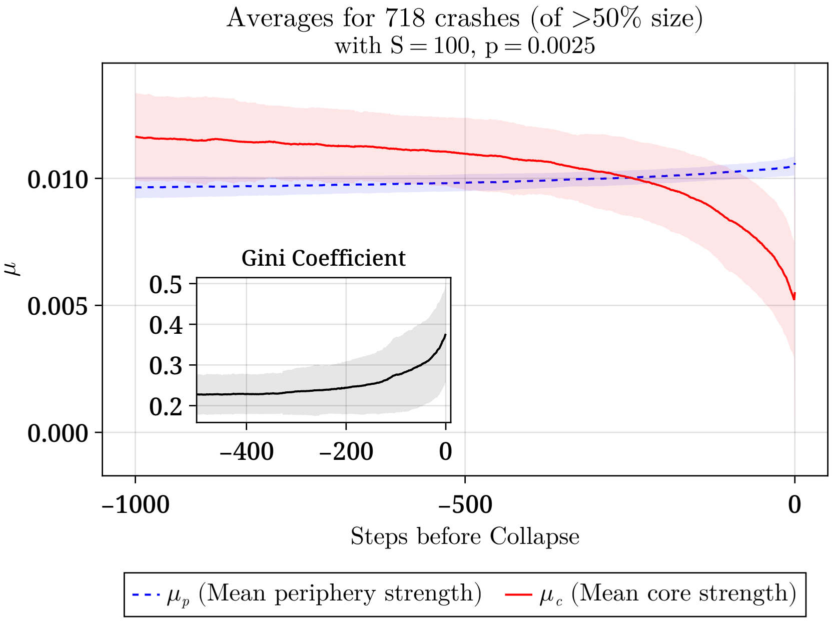

Figure 1: Power shift and rising inequality as precursors to crashes. Time is shown in units of graph updates on the x-axis, where zero denotes the crash time. The y-axis shows quantities averaged over 718 runs each ending in a crash. In a given run, and at a given time, μ_c (μ_p) is the average strength of the core (periphery) nodes of the graph at that time. The origin of time is shifted for each run such that the crash in the run happens at time zero. The red solid curve (blue dashed curve) shows the average of μ_c (μ_p) as a function of time across 718 runs. The blue dashed curve intersecting and going above the red solid curve nearer the crash is evidence of the power shift from core to periphery. The graph inset plots the Gini coefficient as a function of time averaged over the 718 runs and shows rising inequality before collapse. The pink, blue and grey shaded bands represent one standard deviation of μ_c, μ_p and Gini coefficient respectively across the 718 runs. Parameter values for all runs are $S = 100, p = 0.0025$.