

# Exploring Multi-Opinion Dynamics in $q$ -Voter Models

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## Extended Abstract

**Motivation.** Understanding opinion formation in social systems has been a central topic in sociophysics and complex systems research over the past two decades [1–5]. While classical voter and  $q$ -voter models primarily consider binary opinion states, real-world social dynamics often involve more than two competing positions, including moderate or neutral stances. Three-state and multi-opinion models provide a richer framework to investigate polarization, coexistence, and symmetry-breaking phenomena [6–8].

The present study addresses the following research questions:

1. How does extending the  $q$ -voter model to three opinion states modify the collective dynamical behavior?
2. What qualitative dynamical regimes emerge from the interplay of conformity, independence, and external influence?
3. Under which conditions can moderate opinions persist or dominate in a polarized environment?

Answering these questions is important for advancing theoretical understanding of nonlinear collective dynamics and for providing stylized insights into polarization and moderation in real social systems.

**Approach and Methodology.** We consider a three-state extension of the  $q$ -voter model in which agents may adopt one of three discrete opinion states. The dynamics incorporate group conformity (via unanimous panels of size  $q$ ), independent behavior with probability  $p$ , and directional external influence.

The macroscopic behavior is analyzed using a mean-field formulation that yields a system of nonlinear differential equations governing the population fractions of each opinion state. Fixed points and their stability are investigated using linear stability analysis of the Jacobian matrix.

To complement the analytical approach, agent-based Monte Carlo simulations are performed on representative network structures. These simulations allow us to examine finite-size effects, transient dynamics, and qualitative phase-space behavior beyond mean-field approximations.

The combination of analytical and computational methods provides a consistent framework to explore emergent collective regimes in multi-opinion systems.

**Results.** Our analysis reveals that extending the  $q$ -voter framework to three states generates qualitatively richer dynamical structures compared to binary models. Depending on parameter configurations, the system exhibits:

- Consensus states dominated by one opinion,
- Coexistence regimes involving multiple stable fractions,
- Asymmetric configurations induced by external influence, and
- Scenarios in which moderate opinions remain dynamically relevant.

The results indicate that the presence of a third state significantly modifies the structure of attractors in phase space. Both conformity strength (controlled by  $q$ ) and independence probability  $p$  play central roles in shaping convergence patterns and stability properties.

These findings address the posed research questions by demonstrating how additional opinion states alter collective outcomes and by identifying mechanisms that may support moderation in polarized systems.

**Conclusions and Outlook.** We have presented a multi-opinion extension of the  $q$ -voter model that captures richer collective dynamics than traditional binary formulations. The interplay between conformity, independence, and external influence leads to diverse qualitative regimes, including coexistence and asymmetry.

Ongoing work focuses on systematic phase-space classification, bifurcation analysis, and exploration of heterogeneous and complex network topologies. By the time of the conference, additional results on stability structure and numerical phase portraits are expected to be completed.

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