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Exact prime density reproduced through resonant tunneling across a double barrier system

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Extended Abstract

Motivation: Complex systems often exhibit irregular [1,2] yet statistically structured spectra that emerge from simple underlying rules. Prime numbers represent one of the most fundamental examples of such complexity; their global distribution follows smooth asymptotic laws such as the Prime Prime number theorem, while their local spacing is highly irregular. Understanding whether such arithmetic complexity can emerge from physical transport processes is an open, interdisciplinary problem that bridges number theory and quantum physics. Previous approaches [3] have explored prime related spectra using fractal potentials or holographic optical traps, but these methods often suffer from experimental limitations including fluctuating optical power. This motivates the search for a simpler and more robust physical platform. In this work, we ask whether prime number statistics can emerge as resonance spectra of a structured transport system, and whether local prime density and gap growth can be reproduced without explicitly encoding primes (see schematic diagram in fig.1). We demonstrate that a solid-state system based on resonant tunnelling provides such a platform, enabling the reproduction of prime numbers as energy eigenvalues. Establishing a physical realization of prime statistics is important not only for deepening the connection between number theory and quantum mechanics, but also for enabling novel physical operations relevant to computation and complex system modeling.

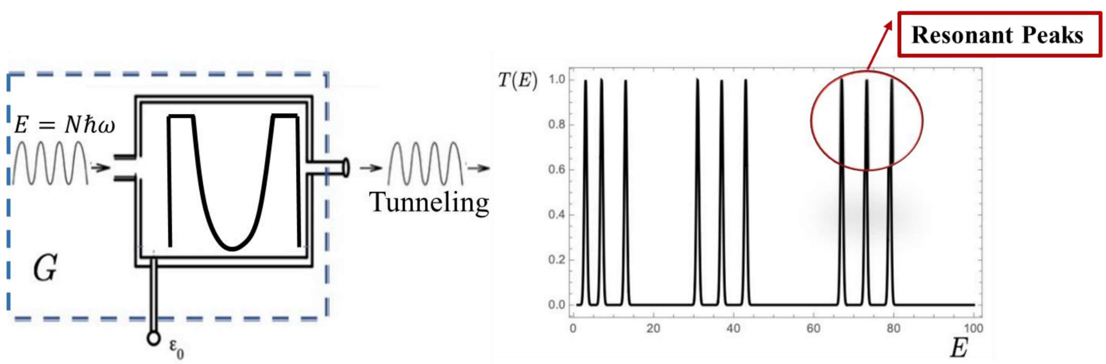
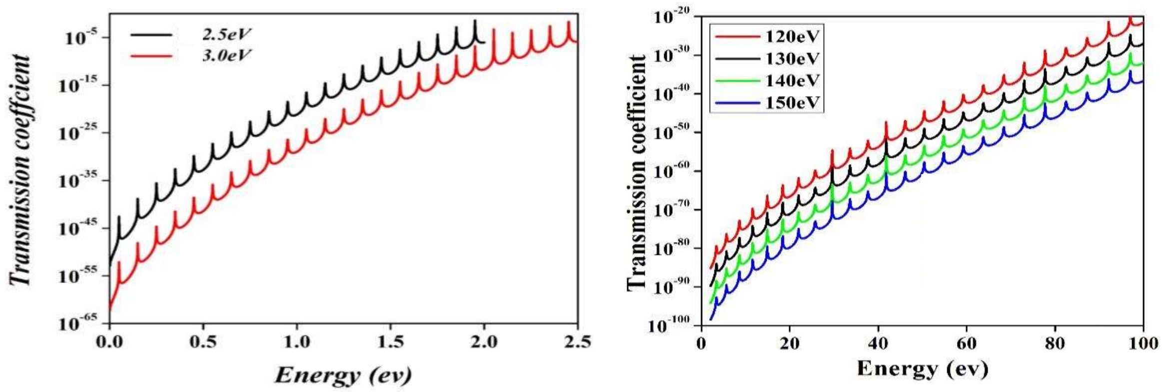


Figure-1: Schematic diagram of our approach.

Approach and Methodology: The study proposes a solid-state experiment utilising a double-barrier system (DBS) in which resonant tunnelling energies are designed to correspond to the prime numbers. The methodology bifurcates the energy spectrum into two distinct components: a smooth part, representing the local average prime spacing and a fluctuating part which acts as a perturbation to achieve exact prime spectra. To determine the smooth potential $V(x)$ for the well region, a semi-classical quantum condition is applied and inverted to form Abel's integral equation. The density of states is calculated using the prime counting function [4], based on Riemann's refined estimation. Transmission coefficients across the DBS [5] are then computed using the transfer matrix approach combined with finite element method, allowing precise identification of resonance peaks. As a benchmark to validate the transport-based spectral extraction, the methodology was first applied to a harmonic oscillator potential. This produced uniformly spaced resonances corresponding to whole numbers (offset by a constant zero-point energy), confirming the reliability of the numerical approach before its application to the more complex prime-based potential.

Result: The resonant tunnelling spectrum of the DBS exhibits a sequence of resonance peaks whose counts in fixed energy intervals exactly match the number of primes in corresponding intervals of the number line. Validation using the quadratic SHO potential produced resonant peaks (see fig. 2a)) that perfectly matched the expected energy levels $\varepsilon_n = (n + \frac{1}{2})\hbar\omega$. In the case of the prime potential, the model yielded 25 resonant states between energies 2 and 100, exactly matching the first 25 primes (fig. 2 b). Testing at higher ranges, such as 5100-5200, continued to match the prime spectrum exactly, yielding 11 resonant states (see fig-5). These results answer the primary research question by confirming that the smooth potential in a DBS can accurately represent the coarse-grained properties of prime numbers.



(a) (b)

Figure 2: Transmission coefficient vs energy in (a) SHO and (b) prime potential. Colored lines represent different barrier heights.

Conclusion and Outlook: The proposed Double Barrier System (DBS) model establishes a robust framework for mapping the distribution of prime numbers, a cornerstone of mathematical complexity, onto physical resonant tunnelling energies. Our results demonstrate that a structured solid-state system can exactly capture the global network of prime densities and average gap growth across multiple energy scales. By reproducing these statistical laws through a semi-classical potential inversion, we provide a physical realisation of the smooth asymptotic behaviour inherent in complex number-theoretic sequences. Future research will investigate the fluctuating part of the potential, seeking the specific local perturbations required to shift these smooth densities into exact, discrete prime eigenvalues. This transition from a smooth statistical average to a precise, irregular spectrum represents a classic problem in the physics of complex systems. We expect this methodology to catalyse practical solid-state experiments, offering a new hardware-based approach to exploring the physical manipulation of prime-based spectra and their applications in secure communication and quantum complexity theory.

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