Dynamical analysis of a reservoir computer

Keywords: Machine learning, Reservoir Computing, Non-linear dynamics, Dynamical transitions, Explainable AI

Extended Abstract

Motivation. Machine learning methods are increasingly used to model complex dynamical systems and to anticipate critical transitions from data. Reservoir computing (RC), a recurrent neural-network framework in which only the output layer is trained, has proven particularly effective for forecasting nonlinear time series [1] and predicting bifurcations when the system parameter is provided as an additional input [2]. While the predictive success of RC has been widely demonstrated, the dynamical mechanisms that enable these predictions remain largely unexplored. In particular, previous work has mainly focused on the accurate reconstruction of attractors, leaving open the question of how data-driven models internally reproduce the mechanisms underlying different types of dynamical transitions. Understanding this mechanism is important both for improving machine-learning algorithms and for building reliable tools to anticipate catastrophic transitions such as bifurcations or crises in natural and engineered systems.

Approach and Methodology. To uncover the internal dynamics responsible for prediction in RC, we analyze the trained reservoir as a discrete-time dynamical map and perform bifurcation analysis of this map. In the first work [3], we consider systems undergoing Hopf bifurcation and train a parameter-aware RC using time series generated for parameter values prior to the transition. By studying the eigenvalues and bifurcation structure of the resulting reservoir map, we examine how the trained model reproduces the transition. In the second work [4], we extend this analysis to discontinuous transitions by investigating boundary and attractor-merging crises using representative systems such as the logistic and Gauss maps. The trained reservoir map is projected onto a lower-dimensional space, allowing us to analyze the underlying bifurcation structure and compare the crisis mechanism and statistical properties with those of the original dynamical systems.

Results. Our analysis reveals that the trained reservoir behaves as a dynamical system whose bifurcation structure mirrors that of the target system. For systems undergoing Hopf bifurcation, the reservoir map undergoes a Neimark–Sacker bifurcation, and the critical parameter value of the map lies in close proximity to that of the original continuous-time system. Eigenvalue analysis further shows that successful training is characterized by consistent eigenvalue arguments that match the dominant frequencies of the underlying dynamics [3]. For discontinuous transitions, the reservoir map reproduces both boundary and attractor-merging crises. Despite being dynamically distinct from the original system, the trained reservoir exhibits the same mechanism of attractor collision with unstable fixed points and reproduces the correct scaling exponent governing transient lifetimes near the crisis [4]. These results demonstrate that RC does not merely reproduce trajectories but internally reconstructs the dynamical mechanisms responsible for critical transitions.

Conclusions and Outlook. This work provides a dynamical-systems interpretation of how reservoir computing predicts both continuous and discontinuous transitions. By analyzing the bifurcation structure of the trained reservoir map, we show that RC captures the underlying mechanisms of Hopf bifurcations and crises. These insights contribute to a deeper understanding of how machine-learning models encode dynamical behavior and may guide the development of more reliable data-driven tools for anticipating critical transitions in complex systems.

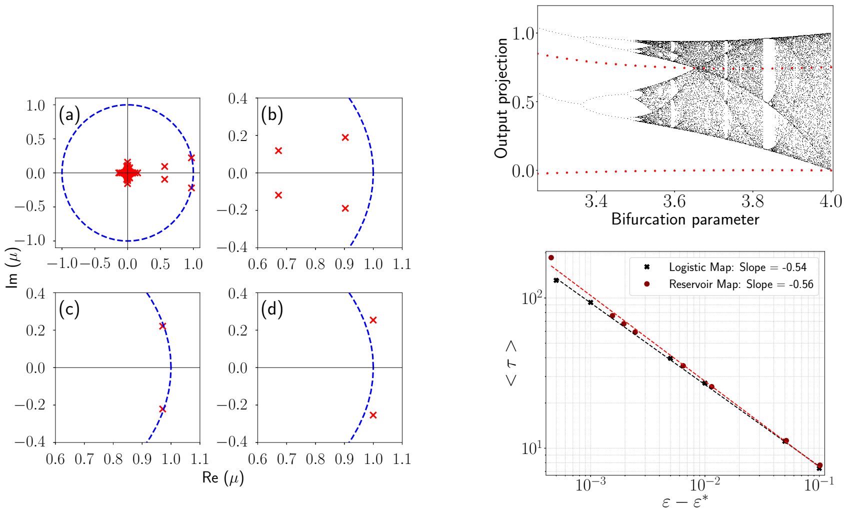

Figure 1: Figure on the left shows eigenvalue crossings with the variation of bifurcation parameter in the RC depicting Neimark–Sacker bifurcation in trained RC map when the input system undergoes Hopf bifurcation. Figure on the right (top) shows the collision of unstable fixed points (red) with the chaotic attractor depicting the crisis mechanism of RC map trained on logistic map time series. Bottom right figure shows the reproduction of scaling exponent during boundary crisis.

References

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