

Percolation Theory-Based Analysis of Sloped Pedestrian Networks

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Extended Abstract

Motivation. Urban populations have grown rapidly in recent decades. According to [1], the number of people living in cities with more than 50,000 inhabitants increased from 1.5 billion in 1975 to 3.5 billion in 2015 and is projected to reach 5 billion by 2050. Despite this growth, access to urban amenities and transportation infrastructure remains uneven across urban areas, with significant inequalities in accessibility observed across different urban populations and neighborhoods [2,3]. Considering that aging and disabled populations are also increasing alongside overall urban populations, the equitable distribution of urban transportation and infrastructure has become an increasingly important issue for inclusive urban planning [4]. These trends highlight the growing need for quantitative approaches to evaluate accessibility and equity in urban systems.

However, pedestrian-oriented accessibility analyses remain limited, particularly those incorporating slope, which can substantially influence walking of accessibility. To address this gap, this study incorporates road slope into a pedestrian network and uses a percolation-based approach to quantify accessibility to everyday amenities represented by Points of Interest (POIs). POI-based measures effectively capture access to daily necessities [5], enabling a more realistic assessment of how network structure translates into lived accessibility.

Within this framework, percolation theory is used to examine how slope-induced constraints restructure pedestrian networks and alter accessibility to essential urban destinations, identifying critical thresholds of network breakdown in cities with diverse mobility needs and complex topography.

Approach and Methodology. Pedestrian road networks were constructed from OpenStreetMap (OSM) data, retaining only walkable road segments (edges) while excluding motorways and other non-pedestrian links. Road slopes were calculated using digital elevation models (DEMs) by extracting elevation values at the endpoints of each road segment and the slope angle was computed from the elevation difference and segment length using the arctangent function. Percolation analysis was then performed by progressively removing edges that exceed given slope thresholds to examine how the overall connectivity of the urban pedestrian network changes. To examine how slope-induced percolation patterns alter access to everyday destinations, Points of Interest (POIs) extracted from OpenStreetMap (OSM) were integrated into the network. Shortest path distances from the centroids of 1km hexagonal grids to POIs were computed on pedestrian networks filtered by different slope thresholds. Accessibility changes were quantified as the ratio between the shortest-path distance in the slope-constrained network and that in the full network as *Detour Index*($DI_{threshold}$) = $\frac{D_{threshold}}{D_{all}}$, where D_{all} represents the corresponding distance in the full network and $D_{threshold}$ denotes the shortest-path distance in the slope-constrained network. A higher DI indicates greater detour or reduced accessibility caused by slope-induced constraints in the pedestrian network

Results. Slope distributions in urban road networks exhibit a heavy-tailed distribution, indicating that most streets have gentle slopes($0^\circ \sim 2^\circ$) while a small number of segments are

relatively steep. By gradually increasing the slope threshold and examining the Largest Connected Component (LCC), we identified distinct transition points, where network connectivity changes abruptly. The transition point is approximately 1° in Seoul, 1.5° in New York, 2° in Busan and 3° in San Francisco. The results show that beyond the critical slope threshold, pedestrian network fragmentation increases, leading to a rapid decline in the reachable urban area. For the accessibility analysis, the detour index (DI) was calculated using Points of Interest (POIs) centered on cultural facilities. For each urban area, DI values were compared between DI_{no_avoid} and DI_{only_easy} and under the constrained-slope condition ($D_{threshold}$), DI varied across cities. San Francisco showed the largest change, with the median detour index increasing by about 0.75 (1.15→1.90). Busan and Seoul also showed moderate increases in the median (Busan: 1.08→1.41, +0.33; Seoul: 1.00→1.29, +0.29). In contrast, New York and Tokyo maintained a median DI of 1.00, with only slight increases in the mean (New York: +0.13; Tokyo: +0.09), suggesting that slope constraints affected only a small subset of routes.

Conclusions and Outlook This study shows that incorporating road slope into percolation-based network analysis reveals how slope conditions influence the spatial extent of pedestrian accessibility. The findings suggest that slope distribution can reshape urban mobility ranges and consequently affect accessibility. This approach empirically measures the influence of road slope on urban mobility and explores its potential as an indicator of mobility characteristics across cities.

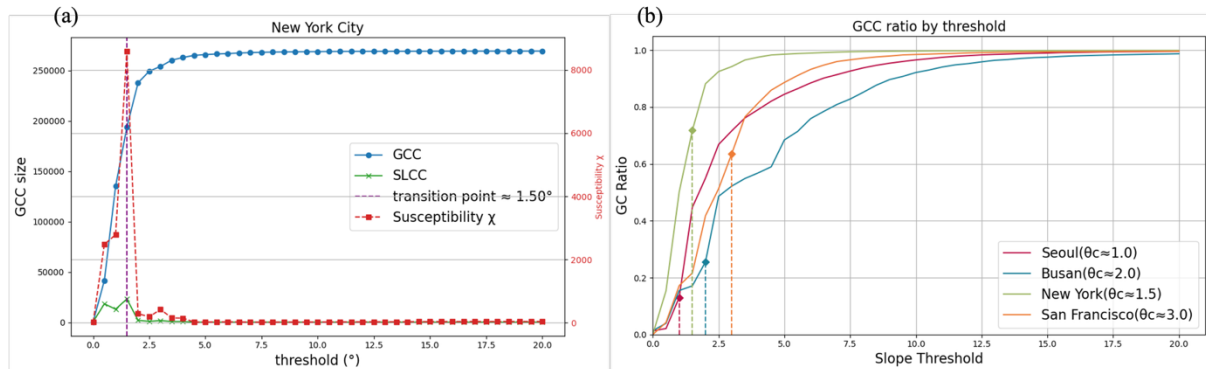


Figure 1. **Percolation analysis of the New York City and GCC(Giant Connected Component) ratio with threshold.** The size of the largest connected component (LCC), second-largest connected component (SLCC), and susceptibility (χ) are shown, with an estimated transition point at approximately 1.5° (a). San Francisco exhibits the highest transition point at around 3° , while Seoul in South Korea shows the lowest transition point at approximately 1° (b).

References

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