Network-Based Characterization of Laminar Flow Using Complex Network Theory: A Data-Driven Analysis of Skin Friction and Heat Transfer

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Extended Abstract

Motivation. Understanding the interaction between governing physical parameters in laminar fluid flow remains challenging when multiple effects such as unsteadiness (A), magnetic field strength (M), porosity (K), and thermal properties are simultaneously considered. Traditional analytical and numerical approaches typically examine these parameters independently or through low-dimensional sensitivity analyses, limiting the ability to capture global interdependencies across multidimensional parameter spaces. This study investigates whether complex network theory can provide a structured framework for characterizing these interactions by representing parameter combinations as nodes and defining similarity relationships based on observable flow quantities. Specifically, we examine whether similarity networks constructed from the skin friction coefficient ($-f''(0)$) and the local Nusselt number ($-\theta'(0)$) can reveal physically meaningful flow regimes. Establishing such a framework would introduce a network-driven diagnostic tool for identifying stable, transitional, and extreme flow behaviors in laminar systems.

Approach and Methodology. The study considers two-dimensional, laminar, incompressible flow over an unsteady stretching sheet. The governing partial differential equations are reduced to coupled nonlinear ordinary differential equations via similarity transformations. Tabulated numerical results of the skin friction coefficient and local Nusselt number are collected under varying conditions of unsteadiness (A), magnetic parameter (M), Prandtl number (Pr), heat source parameter (δ) and porosity (K). Each unique parameter combination forms a node in a similarity network. Euclidean distance computed from normalized values of $-f''(0)$ and $-\theta'(0)$ is used to quantify similarity between nodes. The dataset consists of $N = 20$ distinct parameter combinations, each forming a node in the similarity network. K-means clustering ($k = 5$) is applied to partition the dataset, with cluster validity assessed using silhouette analysis. A similarity graph is then constructed by connecting nodes whose pairwise distance falls below a threshold of 0.7. Network visualization is performed using a force-directed spring layout, with nodes color-coded by cluster membership and edge weights representing similarity strength.

Results. The analysis reveals five distinct clusters corresponding to physically interpretable flow regimes. Higher values of unsteadiness (A) are observed to correspond to increased shear and enhanced heat transfer within the analyzed dataset, whereas strong magnetic effects (M) introduce shear resistance and modify thermal gradients. Variations in Prandtl number and heat source parameter influence thermal boundary layer thickness, thereby altering heat transfer rates. The clustering configuration achieved an average silhouette coefficient exceeding 0.65, demonstrating satisfactory inter-cluster separation and intra-cluster compactness. The resulting network topology highlights densely connected central nodes representing stable flow regimes, while sparsely connected peripheral nodes indicate transitional or extreme parameter combinations. These findings demonstrate that similarity-based network construction preserves known analytical trends while providing additional structural insight into regime transitions.

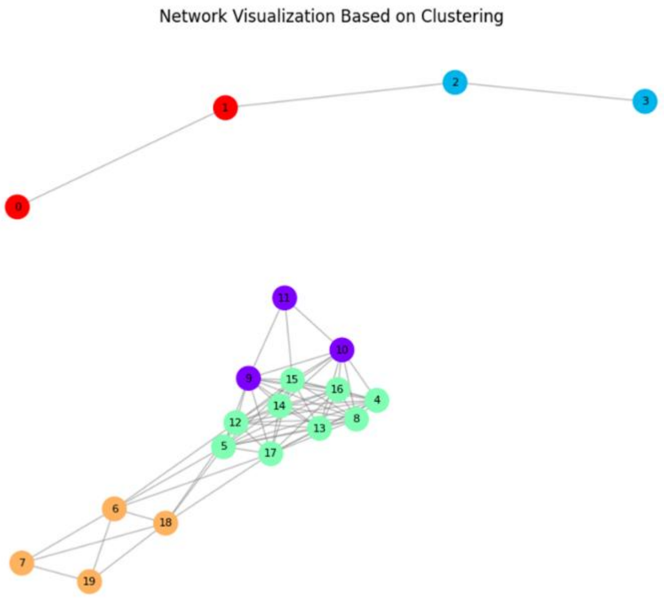

Figure 1. Similarity network of laminar flow parameter combinations (A, M, Pr, δ, K). Edges represent Euclidean similarity based on ($-f''(0), -\theta'(0)$) with threshold 0.7.

Conclusions and Outlook. This work establishes a network-based framework for interpreting laminar flow data through similarity graph analysis. By transforming multidimensional parameter interactions into an analyzable network structure, the method complements classical fluid dynamic analysis with topological insight. Future work will extend this framework to transient and nonlinear regimes, including turbulent flows, where scale-free or community-driven structures may emerge. Network metrics such as centrality and clustering coefficients will also be explored as predictive features in data-driven modeling and flow optimization strategies.

References

[1] Iacobello, G., Ridolfi, L., & Scarsoglio, S. (2021). A review on turbulent and vortical flow analyses via complex networks. *Physica A: Statistical Mechanics and its Applications*, *563*, 125476.

[2] Ikotun, A.M., Ezugwu, A.E., Abualigah, L., Abuhaija, B. & Heming, J. 2023. K-means clustering algorithms: A comprehensive review, variants analysis, and advances in the era of big data. *Information Sciences* 622: 178–210.

[3] Nair, A. G., Douglass, S. B., & Arya, N. (2023). Network-theoretic modeling of fluid–structure interactions. *Theoretical and Computational Fluid Dynamics*, 37(6), 707–723.

[4] Taira, K., & Nair, A. G. (2022). Network-based analysis of fluid flows: Progress and outlook. *Progress in Aerospace Sciences*, *131*, 100823.