Dynamics-based renormalization group for complex networks

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Keywords: complex networks; multiscale dynamics; renormalization group; coarse-graining; dynamical universality classes

Extended Abstract

Motivation. Renormalization group ideas provide a principled way to connect microscopic interactions with emergent macroscopic behavior[1]. While this perspective has been highly successful in statistical physics, its extension to complex networks remains challenging. Real-world networks are typically heterogeneous[2], often display small-world connectivity[3], and generally do not possess a natural geometric embedding[4]. These features make it difficult to define coarse-graining procedures that are both systematic and broadly applicable. At the same time, the effective behavior of a networked system is shaped not only by its topology but also by the dynamics taking place on that topology[5]. This raises a central question: how can one construct reduced descriptions of complex networks that preserve both structural organization and dynamical relevance across scales?

Approach and Methodology. We propose a dynamics-based renormalization framework in which coarse-graining is guided by a local linear representation of the network dynamics near a reference state. The corresponding operator provides a compact description of effective interactions and helps identify collective degrees of freedom that remain important at larger scales. In contrast to purely structural reduction rules, this strategy incorporates dynamical consistency directly into the renormalization process. The resulting procedure is designed to generate reduced network descriptions that remain informative not only in terms of connectivity patterns but also in terms of the functional behavior supported by the original system. More generally, the framework establishes a systematic link between dynamical representations and coarse-grained network models without requiring a predefined metric space or a narrowly specialized aggregation rule.

Results. We evaluate the framework on representative synthetic and empirical networks and find that the renormalized systems retain major large-scale properties of the originals. In particular, the reduced networks preserve important statistical characteristics of network organization while also maintaining key dynamical signatures related to stability and propagation. These results suggest that the method can produce meaningful compressed descriptions without relying exclusively on structural similarity. In addition, the renormalization flows generated by the procedure reveal interpretable multiscale organization and help distinguish different classes of networked systems according to their coupled structural and dynamical features. This indicates that the framework can serve as a useful basis for comparing heterogeneous systems within a common multiscale setting.

Conclusions and Outlook. Our findings support the idea that incorporating dynamical information directly into coarse-graining can strengthen renormalization approaches for complex networks. By combining structural and functional considerations within a unified framework, the proposed method provides a promising route toward multiscale analysis of heterogeneous networked systems. Ongoing work is extending this framework to broader classes of nonlinear and adaptive dynamics, as well as to additional real-world applications. Future studies will further examine how such renormalized descriptions can be used to characterize effective macroscopic behavior, robustness, and universality across diverse complex systems.

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