

# Reconstructing connection weights and topology of Directed Networks from Noisy node Dynamics

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One of the major ultimate goals in science is to achieve a logical understanding of the phenomenon and, hopefully, to uncover the basic principles behind it and be able to make predictions for the system. Machine learning may be very helpful in summarizing these vast data volumes and classifying them into patterns or structures, but a knowledge-based approach to analyzing the data can do a much better job of understanding the reasoning behind the data and of modeling the system or discovering scientific laws buried in the vast data mines. The relations between different entities in the dataset can often be described in terms of a network consisting of nodes and links. It is a highly non-trivial task to construct the connection topology and link strengths of a network that can faithfully represent the underlying interactions among the network elements. Often, the rules for assigning a connection and its strengths between two nodes lack rigorous scientific reasons or are not well-justified. In addition, the connections or their strength are usually not known, but the behavior of the nodes can be monitored or measured. One of the challenging inverse problems is to retrieve the network connection weights or node-node interactions from the passively observed dynamics of the nodes, which are often accessible or measurable.

We consider coupled network dynamics under uncorrelated noises that fluctuate about the noise-free long-time asymptotic state. Our goal is to reconstruct the connection weights and topology of a directed network solely from time-series data of node dynamics [1-5]. Using the stochastic Force Inference method[6] with a simple natural choice of linear polynomial basis, we derive a reconstruction scheme of the connection weights and the noise strength of each node. Explicit simulations for directed and undirected random networks with various node dynamics are carried out to demonstrate the good accuracy and high efficiency of the reconstruction scheme[7]. We further consider the case when only a subset of the network and its node dynamics can be observed[8], and it is demonstrated that the directed weighted connections among the observed nodes can be easily and faithfully reconstructed. In addition, we propose a scheme to infer the number of hidden nodes and their effects on each observed node. Simulations illustrate the accuracy of these results.

**Keywords:** Noisy network dynamics, network reconstruction, stochastic force inference, data mining

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