Complex dynamics of the Hindmarsh-Rose Neuron Model with Blue-Sky Catastrophe under a Magnetic Field and Noise

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Abstract

Motivation. Neuronal activity is influenced not only by intrinsic nonlinear dynamics but also by external physical factors such as electromagnetic interactions and stochastic fluctuations. The Hindmarsh-Rose (HR) neuron model associated with blue-sky catastrophe is widely used to study complex firing patterns, including bursting and chaotic bifurcations. However, real neuronal systems operate in noisy environments due to channel noise and synaptic variability. Therefore, understanding how magnetic-field-induced feedback interacts with stochastic noise is important for describing realistic neuronal dynamics. In this work, we study the combined influence of magnetic flux coupling and stochastic perturbations on the HR neuron model.

Approach and Methodology. We extend the HR model by introducing a magnetic flux variable that couples nonlinearly with the membrane potential. Two magnetic coupling functions are considered: a polynomial interaction $W(\phi) = \alpha + 3\beta\phi^2$ and a nonlinear saturating interaction $W(\phi) = -\tanh(\phi)$. Stochastic effects are incorporated through a noise term with intensity ϵ. The system dynamics are analyzed using inter-spike interval (ISI) bifurcation diagrams with respect to the external current I_{ext} for different noise levels. To further characterize the stability of the dynamics, we compute the largest Lyapunov exponent of the model system and examine time series behavior and phase-plane analysis.

Results. We find that magnetic feedback significantly modifies the neuronal firing patterns and bifurcation structure. In the deterministic case ($\epsilon = 0$), clear transitions from periodic spiking to chaotic bursting are observed as I_{ext} increases. With weak noise strength ($\epsilon = 0.001$), the similar bifurcation structure remains visible with slightly blurred nature due to the fluctuations in the dynamics. However, in the strong noise strength case ($\epsilon = 0.1$), the bifurcation branches become scattered from a symmetric line driven by fluctuations, indicating increased in variability and coexistence of firing patterns. Overall, the study demonstrates that magnetic coupling and stochastic fluctuations jointly shape the stability and complexity in the neuronal dynamical patterns in the single HR system.

Conclusions and Outlook. Magnetic-field-induced feedback significantly alters the firing dynamics and bifurcation structure of the Hindmarsh-Rose neuron model, while stochastic noise characterizes the variability in the hidden patterns that can drive noise-induced transitions and promote irregular spiking patterns in the neuronal dynamics. Future studies may extend this framework to neuronal networks and explore different electromagnetic couplings and noise-induced transitions driven by various noise types to better understand their role in realistic neural dynamics, information processing and synchronization in neural systems.

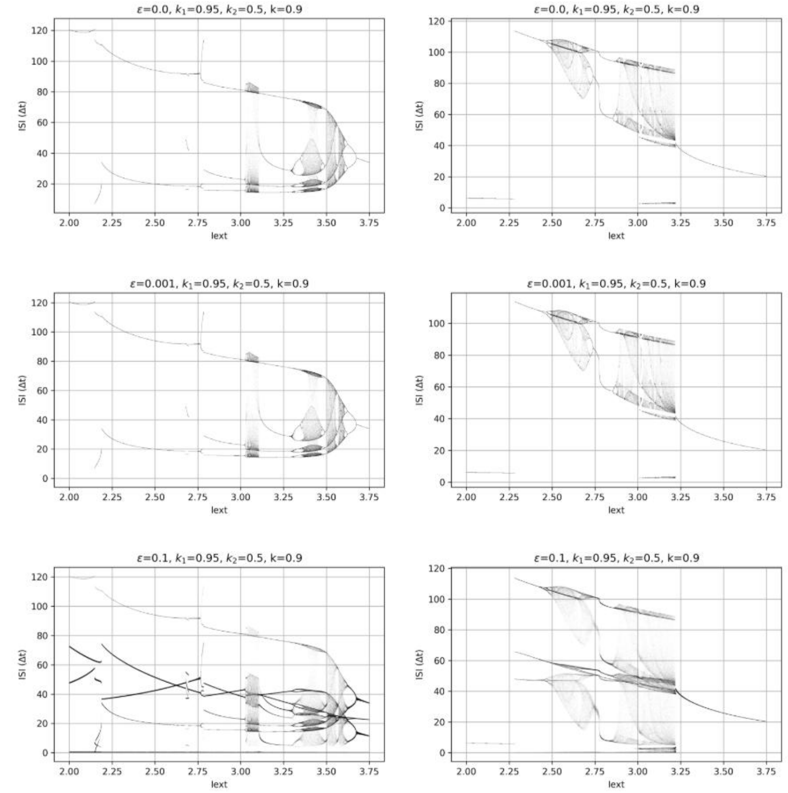

Figure 1. **Bifurcation diagrams of the inter-spike interval (ISI, Δt) versus external current I_{ext} for the Hindmarsh-Rose (HR) neuron model in the presence of magnetic flux and deterministic/stochastic noise.** The panels illustrate how the firing dynamics of the neuron change with variations in external current, magnetic field coupling, and noise intensity. The **left column** corresponds to the magnetic coupling function $W(\phi) = \alpha + 3\beta\phi^2$, while the **right column** corresponds to the nonlinear saturating magnetic interaction $W(\phi) = -\tanh(\phi)$. The **rows represent increasing stochastic noise power**, with $\epsilon = 0.0$ (top row) corresponding to the deterministic case, while $\epsilon = 0.001$ (middle row), and $\epsilon = 0.1$ (bottom row) corresponding to the stochastic cases. Other parameters are fixed at $k_1 = 0.95$, $k_2 = 0.5$, and $k = 0.9$. Each point represents the measured inter-spike interval obtained from the neuronal time series after removing transient dynamics.

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