Weak higher-order interactions enhance synchronization

Riccardo Muolo[1], Hiroya Nakao[2], Marco Coraggio[3]
[1]RIKEN iTHEMS, Saitama, Japan
[2]Scuola Meridionale Superiore, Naples, Italy
[3]Department of Systems and Control Engineering, Institute of Science Tokyo, Japan

Keywords: higher-order networks, synchronization, Kuramoto model, hypergraphs

Extended Abstract

Motivation. Synchronization is a fundamental phenomenon in complex systems, observed across a wide range of natural and engineered contexts [1]. The Kuramoto model [2] provides a foundational framework for understanding synchronization among coupled oscillators, traditionally assuming pairwise interactions. However, many real-world systems exhibit group and many-body interactions, which can be effectively modeled through hypergraphs [3]. Previous studies suggest that higher-order interactions make it harder to reach synchronization, while potentially enriching the dynamics. In particular, Zhang et al. [4] showed that higher-order interactions make the attraction basin of the synchronous state smaller but more robust (deeper). These conclusions were further supported in [5], showing that the critical coupling for synchronization increases, facilitating desynchronization, and in [6], which demonstrated that once synchronization is achieved, it becomes harder to disrupt. These findings motivate two research questions: (i) how do higher-order interactions quantitatively affect the synchronous state ? and (ii) given a limited amount of resources for connectivity of both pairwise and higher-order interactions, what is the optimal combination to enhance synchronization?

Approach and Methodology. To address the first question, we studied Kuramoto oscillators coupled through 2- and 3-body interactions and conducted a detailed numerical analysis on various random hypergraphs. We varied the strength of higher-order interactions and explored different initial conditions, from near-synchronous to incoherent states, in order to quantify changes in the attraction basin and synchronization thresholds. To address the second question, we introduced a constraint on the total budget of interactions (pairwise and higher-order) and analyzed how different allocations within this budget affect synchronization. By comparing purely pairwise, purely higher-order, and mixed interaction structures, we assessed which configurations optimize synchronization under finite resources.

Results. Our study [7] confirms that higher-order interactions enhance synchronization when initial conditions are close to the synchronous state and supports previous findings that higher-order interactions shrink the attraction basin of the synchronous state while deepening it. However, when starting from incoherent states, we find that weak higher-order interactions can enhance synchronization (Fig. 1a). Furthermore, under a limited interaction budget, synchronization is maximized by a combination of pairwise and higher-order interactions, regardless of the relative cost assigned to higher-order links (Fig. 1b).

Conclusions and Outlook. We show that weak higher-order interactions can enhance synchronization and that, under finite connectivity resources, a mixed interaction architecture optimizes synchronizability. These findings provide guidance for the design of engineered systems and offer insight into interaction patterns emerging in natural systems.

References

[1] A. Pikovsky, J. Kurths, and M. Rosenblum. *Synchronization: A Universal Concept in Nonlinear Sciences*. Cambridge University Press, 2001.

[2] Y. Kuramoto. Self-entrainment of a population of coupled non-linear oscillators. In H. Araki (Ed.), *International Symposium on Mathematical Problems in Theoretical Physics*, Springer, 1975, pp. 420–422.

[3] F. Battiston et al. The physics of higher-order interactions in complex systems. *Nature Physics*, 17 (2021), pp. 1093–1098.

[4] Y. Zhang et al. Deeper but smaller: Higher-order interactions increase linear stability but shrink basins. *Science Advances*, 10.40 (2024), eado8049.

[5] R. Fariello and M. A. M. de Aguiar. Third order interactions shift the critical coupling in multidimensional Kuramoto models. *Chaos, Solitons & Fractals*, 187 (2024), 115467.

[6] S. von der Gracht, E. Nijholt, and B. Rink. Higher-order interactions lead to 'reluctant' synchrony breaking. *Proceedings of the Royal Society A*, 480.2301 (2024), 20230945.

[7] R. Muolo, H. Nakao, and M. Coraggio. When higher-order interactions enhance synchronization: the case of the Kuramoto model on random hypergraphs. *arXiv preprint arXiv:2508.10992*, 2025.

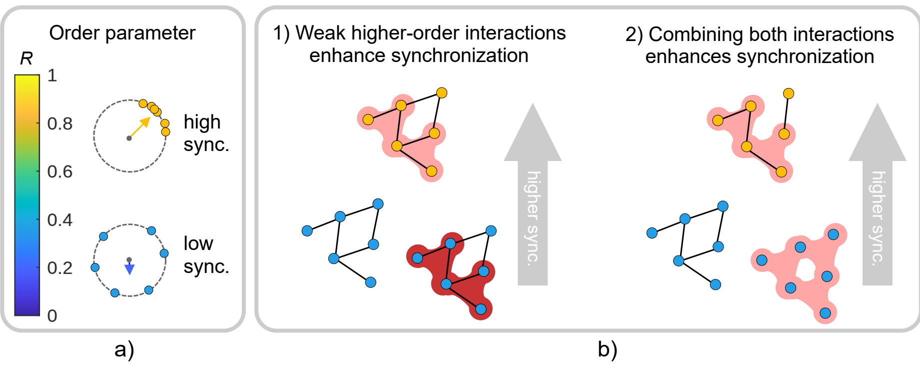

Figure 1: **Main findings of this work:** (1) While higher-order interactions generally hamper synchronization by making the attraction basin of the synchronous state "deeper but smaller", weak higher-order interactions enhance synchronization. (2) Under a finite budget for interactions, regardless of the relative cost of higher-order interactions, the optimal configuration for synchronization always involves a combination of pairwise and higher-order interactions. The colors of the hyperedges give a pictorial representation of the coupling strength, namely red indicates a strong coupling while pink denotes a weaker one.