

Emergence of Metastability in Spatially Embedded Neuronal Networks with Higher-Order Interactions

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Extended Abstract

Motivation. Metastability, a key feature of brain dynamics, is a state that allows neural systems to flexibly transition between partially synchronized configurations. Although higher-order interactions have been widely studied using frameworks such as simplicial complexes and hypergraphs, they are often biologically motivated by mechanisms such as glial-mediated neuronal modulation. However, their role in spatially embedded, distance-dependent networks reflecting realistic brain connectivity remains unclear. Here, we examine how distance-dependent connectivity and effective higher-order interactions shape metastable dynamics in Izhikevich neurons on a Waxman-type network, as understanding these effects is crucial for revealing how spatial structure and complex interactions regulate the balance between integration and segregation in cortical neuronal ensembles.

Approach and Methodology. We consider an ensemble of N Izhikevich neurons [2] randomly distributed in a two-dimensional square domain (100×100). Network connectivity follows a distance-dependent Waxman model [4]:

$$p_{ij} = p_0 e^{-\left(\frac{s_{ij}}{\beta L}\right)}$$

where p_0 is the maximum connection probability, s_{ij} the Euclidean distance between neurons i and j , β the spatial decay parameter that controlling connection range, and L the maximum possible distance within the domain. The neuronal dynamics are governed by

$$\frac{dv_i}{dt} = 0.04v_i^2 + 5v_i + 140 - u_i + I_i, \quad \frac{du_i}{dt} = a(bv_i - u_i),$$

with after-spike reset $v_i \geq 30$ mV: $v_i \leftarrow c$, $u_i \leftarrow u_i + d$. The synaptic input to neuron i is

$$I_i(t) = I_i^{dc} + \sum_{d=1}^D d^{-\alpha} \sum_{j|d_{ij}=d} I_{ij},$$

where I_i^{dc} is external input, d_{ij} is the topological distance, and $I_{ij} = \frac{k}{\sigma} [e^{-2(t-t_j)}(V_0 - V_i)]$, represents the synaptic contribution. V_0 is the excitatory reversal potential, and synaptic conductance k is normalized by the node degree σ . In this framework, the input current to neuron i receives contributions from neurons located at different topological distance d , with interaction strength decaying as $d^{-\alpha}$ [1, 3].

Results. The results reveal rich collective dynamics in the neuronal network under effective higher-order interactions, whereas purely pairwise coupling ($D = 1$) leads to fully synchronized spiking and bursting. Higher-order interactions ($D = 2$) induce partially synchronized network states exhibiting temporal fluctuations and the intermittent formation and dissolution of coherent clusters (Fig.- 1), indicative of metastable dynamics. At low coupling strengths, the system remains phase-locked, while intermediate and strong coupling regimes display pronounced temporal fluctuations and metastable cluster reorganization. The metastability index σ_R quantifies persistent variability in the degree of synchronization, confirming the non-stationary nature of the collective states (Fig. 1).

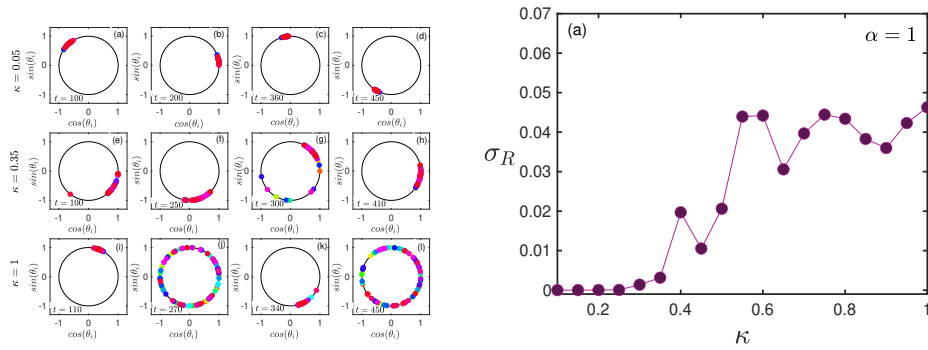


Figure 1: Left Panel: Snapshots of the instantaneous phase distribution of all neurons for different coupling strengths. Right Panel: The metastability index of the network as a function of coupling strength for $D = 2$ and $\alpha = 1$.

Conclusions and Outlook. Higher-order interactions in spatially embedded neuronal networks shape collective dynamics, producing metastable states absent under purely pairwise coupling. These results highlight the pivotal role of spatial connectivity and complex interactions in balancing integration and segregation. Future work will examine the impact of higher-order interactions on metastability and synchronization in mixed excitatory–inhibitory Izhikevich networks.

References

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