Universal Stagnation Dynamics in Quantum Imaginary-Time Evolution

Keywords: Imaginary Time Evolution, Cooling, Ground State Preparation, Stagnation Dynamics, Random Matrix Theory

Extended Abstract

Motivation. Imaginary-Time Evolution (ITE) provides a promising approach for ground-state preparation and solving related optimization tasks, and has recently been revisited in the context of near-term quantum algorithms [2]. More generally, ITE connects cooling dynamics in statistical physics, steepest-descent optimization on Riemannian manifolds [5], and relaxation phenomena in quantum many-body physics. Despite its appealing structure, practical implementations of ITE—ranging from Quantum Monte Carlo [3], Double-Bracket Quantum Imaginary-Time Evolution (DB-QITE) [2], to variational algorithms (such as VarQITE [4])—often display pronounced slowdowns or *stagnations*: extended intervals during which the energy decreases only slowly [6]. While ITE has attracted substantial attention in near-term settings [7], the spectral mechanisms and quantitative timescales governing these slowdowns are not yet well characterized. In particular, a clear account of how spectral geometry (gaps and initial-state overlaps) controls stagnation durations remains incomplete.

Approach and Methodology. Our methodology combines analytical derivations with numerical simulations. We begin by formalizing the stagnation phenomenon: intuitively, the algorithm "appears stuck" when energy decreases so slowly as to be practically indistinguishable from a plateau. Concretely, for a fixed tolerance $\varepsilon > 0$ we define $T_{\mathrm{stag}}(\varepsilon)$ as the total duration over which the energy decay rate is small: $|dE(\tau)/d\tau| \leq \varepsilon$. Then we analytically derive explicit upper and lower bounds for the stagnation time, $T_{\mathrm{stag}}(\varepsilon)$, relying solely on the Hamiltonian spectrum and initial-state overlaps in the energy eigenbasis, as illustrated in Fig. 1(a). Then, we apply Random Matrix Theory (RMT) to these bounds to predict the asymptotic tail distribution of stagnation times for generic classes of many-body Hamiltonians, specifically analyzing Poisson, GOE ($\beta = 1$), and GUE ($\beta = 2$) ensembles. Finally, we numerically evaluate $T_{\mathrm{stag}}(\varepsilon)$ across 8-qubit instances of these random ensembles to verify our scaling predictions.

Results. Combining the stagnation-time bounds with level-spacing statistics yields a prediction for the asymptotic distribution of long stagnation events. For ensembles with Dyson index β, we obtain $P_\beta(T) \sim T^{-(\beta+2)}$ as $T \to \infty$. Thus, increased level repulsion reduces the likelihood of *very long* stagnation times, without implying that small gaps (or long stagnation) are forbidden for any particular instance. Across 8-qubit numerical instances we find (i) stagnation durations that fall between the theoretical bounds and (ii) tail behavior consistent with the $T^{-(\beta+2)}$ prediction and with heavier tails for Poisson than for GOE/GUE, as in Fig. 1(b). To confirm that these findings extend beyond random matrix ensembles to physical systems, we further validate the predicted tail behavior on the Random Transverse-Field Ising Model (RTFIM) [1] in Fig. 1(c).

Conclusions and Outlook. Beyond the specific ensembles studied here, our results provide a quantitative link between *spectral statistics* (gaps and level repulsion) and *algorithmic slowdowns* in ITE-based procedures. Future work will leverage this framework to benchmark stagnation mitigation strategies (e.g., improved initialization or spectral engineering) by comparing how they reshape the gap statistics that control the tail of $T_{\text{stag}}(\varepsilon)$.

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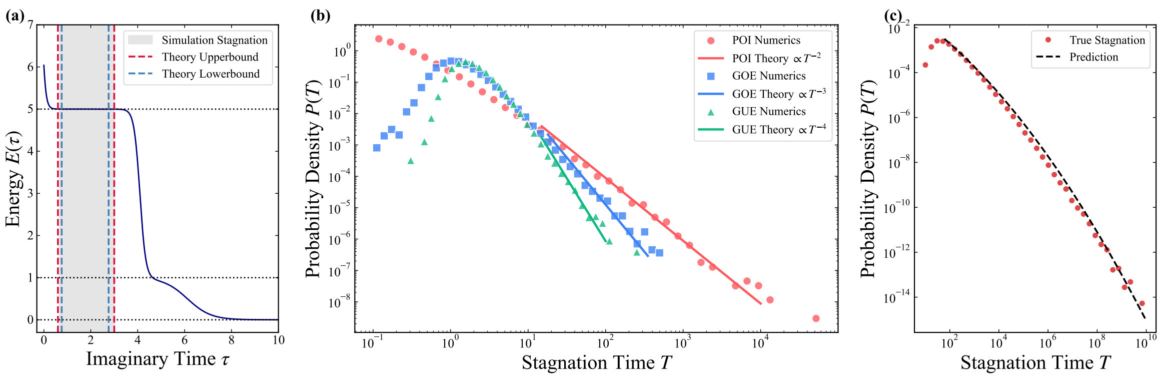

Figure 1: **(a)** Example ITE energy expectation value trajectory $E(\tau)$ exhibiting an intermediate slowdown/stagnation dynamics. Vertical dashed lines indicate the entry/exit-time bounds derived from spectral gaps and initial overlaps. **(b)** Empirical density of stagnation times for Hamiltonians drawn from Poisson, GOE, and GUE ensembles. **(c)** Stagnation time distribution for the ensemble of the RTFIM.