The Geometry of Panic: Multiscale Functional Analysis of Stock Market Dynamics

Keywords: Systemic risk, Complex Financial Network, Topological Data Analysis, Vietoris-Rips Filtration, Ollivier-Ricci Curvature.
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Extended Abstract

Motivation.
Financial markets are complex and non-Euclidean systems. Tradition network filtering systems artificially flatten these relationships, masking structural phase shifts that occur during macroeconomic shock. By treating systemic risk as a geometric constant property, we investigate whether market fragility can be quantified using differential geometry. The objective of this research is to build multiscale topological pipeline to map the structural evolution of the Nikkei 225 during extreme market events.

Approach and Methodology.
To begin analyzing the geometry of the Nikkei 225 constituents, their prices were transformed using Integrated Quasi-Differentiation (IQD). This achieves stationarity while preserving the fractional memory erased by standard log returns. We perform vietoris-Rips filtration on the cross correlation matrix, capturing higher fimensianal interactions that traditioinal methods artificially flattens. By calculating the Ollivier-Ricci Curvature, we perform a topological surgery to remove edges with negative curvatures separating them into their respective communities. To visualize phase transitions, we map the constituent stocks as they transition between communities. This cross-temporal tracking is visualize using an Alluvial Diagram allowing us to identify temporal evolution of topological signatures.

Results.
Applying this pipeline revealed distinct topological signatures for macroeconomic schocks. The COVID-19 crash exhibited total structural fusion, collapsing nearly all constituents into a single comunity, inidicating servere systemic fragility. Coversely, during the Russia-Ukraine confilict instead of a total fusion, several communities is decoupled from the main community. This indicates a less fragile system; these isolated sub-complexes retain independent price dynamics providing for alternative topological pathways during the sell-off.

Conclusions and Outlook.
This pipeline demonstrates that systemic risk is not merely a function of price action but a measurable geometric deformity. While traditional correlation metrics are strictly reactive spiking only *after* a crash has begun, our topological approach isolates the hidden structural decay that makes such contagion possible. By proving that catastrophic liquidity crises and geopolitical shocks possess fundamentally different topological signatures, we establish a new framework for market analysis. Moving forward, this pipeline could serve as a noise-resistant feature extraction pipeline for machine learning models, to transition into a predictive pipeline and potentially building an early warning systems.

References

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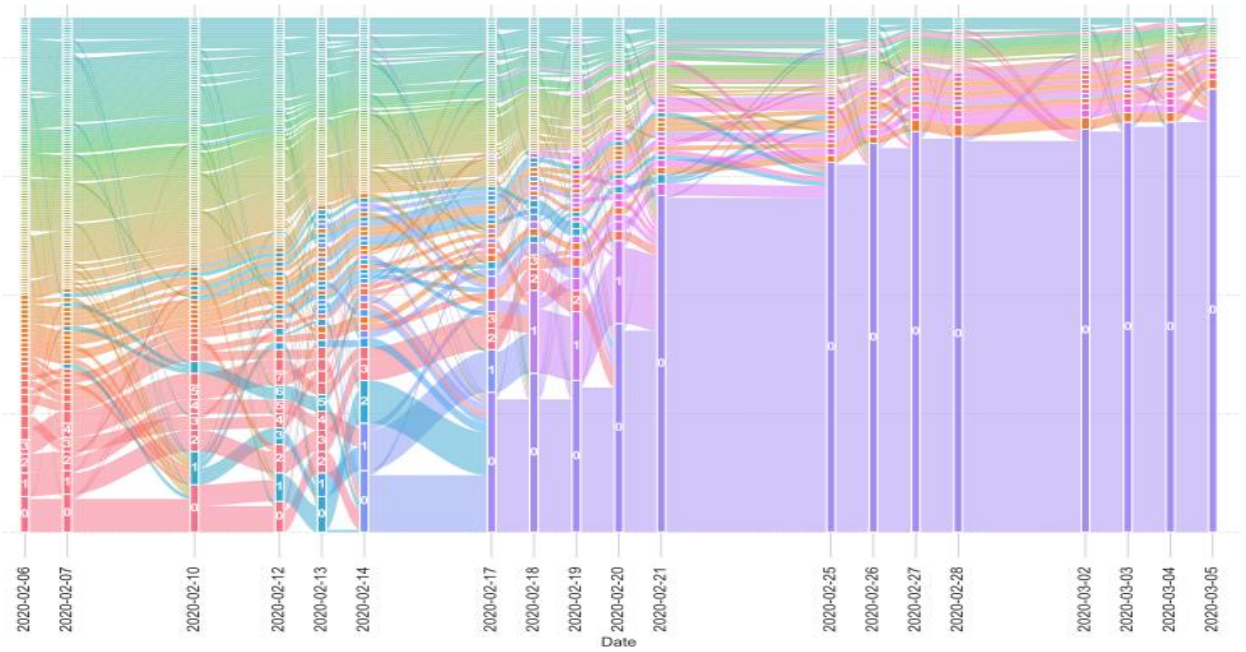

Figure 1 Alluvial diagram of the COVID-19 crash from 6 Feb 2020 (t=509) to 5 Mar 2020 (t=527).

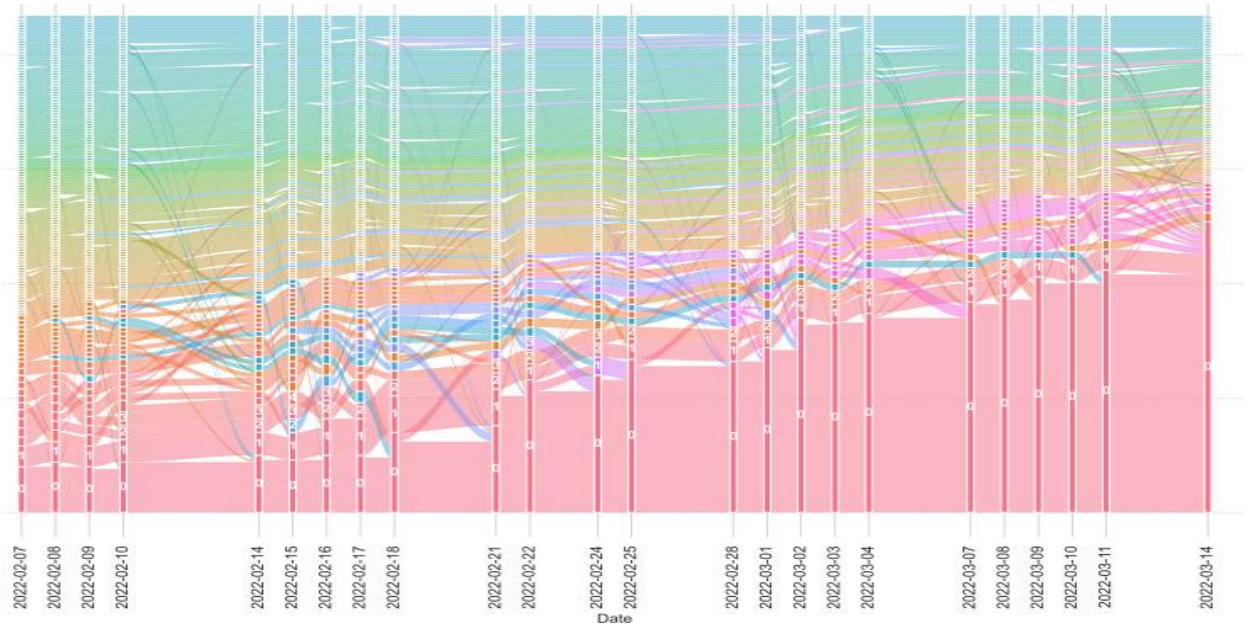

Figure 2 Alluvial diagram of Russian-Ukraine Invasion from 7 Feb 2022 (t=998) to 14 Mar 2022 (t=1021).