

Disassortative Mixing in Decentralized Social Network Due to Reciprocal Links

Keywords: assortativity, heavy-tailed distribution, reciprocal links, network evolution, viral spreading

Extended Abstract

Motivation. Social networks can be represented as graphs and studied quantitatively with tools from network science[1, 2, 3]. Empirical analyses often focus on degree distributions, degree correlations, and spreading dynamics[4, 5, 6]. In the blockchain social network Farcaster[7], we observe negative directed degree assortativity coefficients within our analysis scope and the degree distribution is well fitted by a heavy-tailed form (Fig. 1). The relative ordering of the four directed assortativity measures remains stable as the network grows. This raises a mechanistic question: which local link formation behaviors can generate such stable directed disassortativity in a growing social network?

Approach and Methodology. The research methodology consists of two complementary parts: identifying candidate mechanisms from the empirical network growth and using simulations to quantify how these mechanisms shape network structure. To validate the mechanisms, we measure the out-degree pattern, target-selection preference, and the distribution of reciprocal follow-back probabilities, fit their empirical forms, and encode them as ingredients of the generative model.

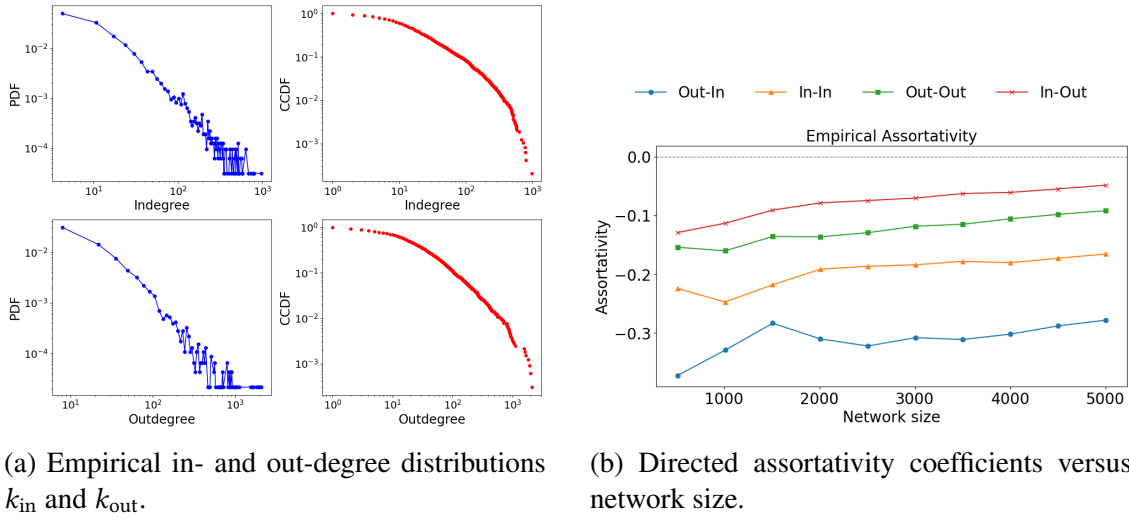
Results. We propose and calibrate a network growth model with three core mechanisms: a heavy tailed out degree distribution, age based preferential attachment, and reciprocal connections. The model reproduces the observed negative directed assortativity coefficients and their stable ordering (Fig. 2a). The circles denote the empirical assortativity values at $N = 5000$, while the points show the model results. The two sets follow the same ordering, with only small deviations. Fig 2b further indicates that reciprocity is the primary control parameter driving the transition of assortative patterns: as reciprocity increases, the coefficients shift from a mixed-sign regime to a uniformly negative regime. Using the calibrated model, directed bond percolation shows that increasing reciprocity lowers the critical transmissibility, making large-scale spreading more likely (Fig. 2c).

Conclusions and Outlook. Our results suggest that reciprocity provides a minimal mechanistic explanation for stable directed disassortativity in Farcaster and that it systematically reduces the percolation threshold. By the time of the conference, we plan to report robustness checks on larger network sizes and a broader parameter range to test the stability of these conclusions.

References

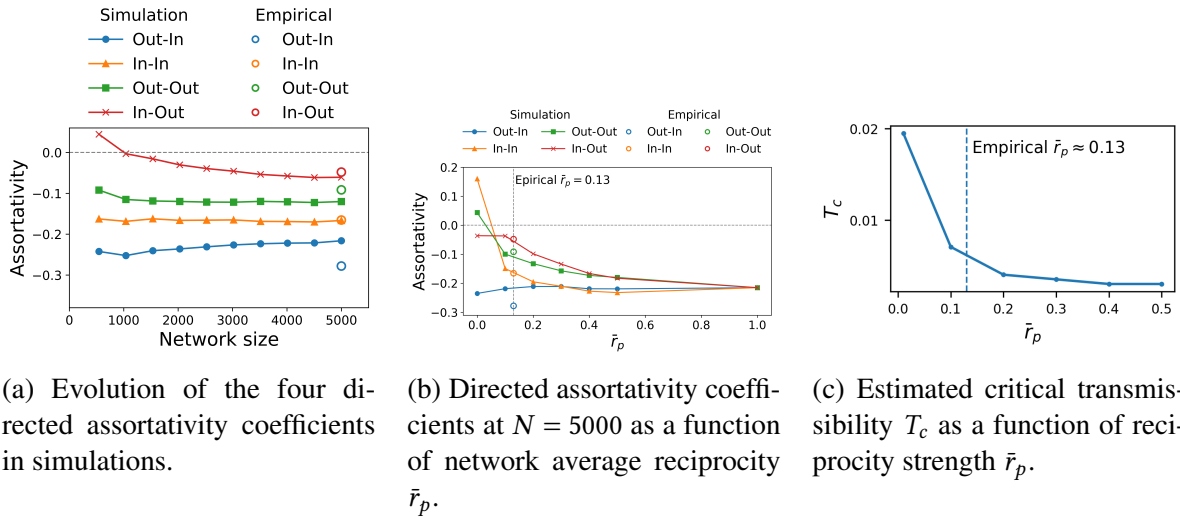
- [1] P. Erdős and A. Rényi. On random graphs i. *Publicationes Mathematicae Debrecen*, 6:290–297, 1959.

- [2] M. E. J. Newman, S. H. Strogatz, and D. J. Watts. Random graphs with arbitrary degree distributions and their applications. *Phys. Rev. E*, 64:026118, Jul 2001.
- [3] Albert-László Barabási and Réka Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.
- [4] M. E. J. Newman. Mixing patterns in networks. *Physical Review E*, 67(2), February 2003.
- [5] Jacob G. Foster, David V. Foster, Peter Grassberger, and Maya Paczuski. Edge direction and the structure of networks. *Proceedings of the National Academy of Sciences*, 107(24):10815–10820, June 2010.
- [6] P. Grassberger. On the critical behavior of the general epidemic process and dynamical percolation. *Mathematical biosciences*, 63(2):157–172, 1983.
- [7] Farcaster Documentation. What is farcaster: Messages, 2025.



(a) Empirical in- and out-degree distributions k_{in} and k_{out} . (b) Directed assortativity coefficients versus network size.

Figure 1: **Empirical patterns in Farcaster.**



(a) Evolution of the four directed assortativity coefficients in simulations. (b) Directed assortativity coefficients at $N = 5000$ as a function of network average reciprocity \bar{r}_p . (c) Estimated critical transmissibility T_c as a function of reciprocity strength \bar{r}_p .

Figure 2: **Simulated assortativity and spreading consequence**